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2ND QUARTERLY TECHNICAL NOTE
STEADY-STATE SHIELDING RESPONSE OF AN IRRADIATED PLASMA

THIS REPORT HAS BEEN PREPARED BY
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GEOPHYSICS CORPORATION OF AMERICA
BEDFORD, MASSACHUSETTS

GCA TECHNICAL REPORT 61-29-A

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AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
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NEW YORK

GEOPHYSICS CORPORATION OF AMERICA
BEDFORD, MASSACHUSETTS

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ABSTRACT

For a fairly broad range of certain nondimensional parameters it is possible to couple RF power incident on an inhomogeneous plasma with the resultant beam attenuation appearing as ohmic heating of the medium. The plasma reacts to this heating with a local rise in temperature and electron density which in turn alters the pattern of heat deposition.(Uncl.)

A closed form solution has been obtained for the plasma response to this heat input using the WKB solution. With an initial exponential distribution of electron density the solution predicts, after the transient response is washed out, the formation of a plasma shield in the form of a progressive invariant wave moving upstream at constant velocity toward the beam source. The shielding velocity and the shape of the wave is specified in terms of basic plasma parameters. (Uncl.)

STEADY-STATE SHIELDING RESPONSE OF AN
IRRADIATED PLASMA

I. INTRODUCTION

We wish to investigate the time response of an inhomogeneous, thermally-ionized plasma irradiated at normal incidence by a beam of electromagnetic flux. A previous study[1] has shown the general features of the reaction:

1. the plasma tends to shield itself with a zone of local electron concentration which rapidly moves upstream towards the beam source,

2. at the same time the electron density gradient of the shielding spike steepens, thus increasing the reflection coefficient of the plasma (see Figures 1 and 2). (Uncl.)

The work cited made use of time-iterated solutions of Maxwell's wave equation to obtain a history of the energy deposition pattern. Although this technique yielded quantitative results, the numerical solutions exacted their usual price, namely a loss of understanding of the underlying physics. (Uncl.)

We present here an analytic closed form solution of the response problem. The only concession is the use of the WKB approximation for the wave equation. The report will specify such hitherto elusive parameters as the transient response time, the steady-state upstream shield velocity, and the shape and slope of the shield. Although first order plasma theory will be used throughout, the entire phenomenon is ~~non~~ linear in character. Maxwell's equations are linear in the field vec-
(Uncl.)

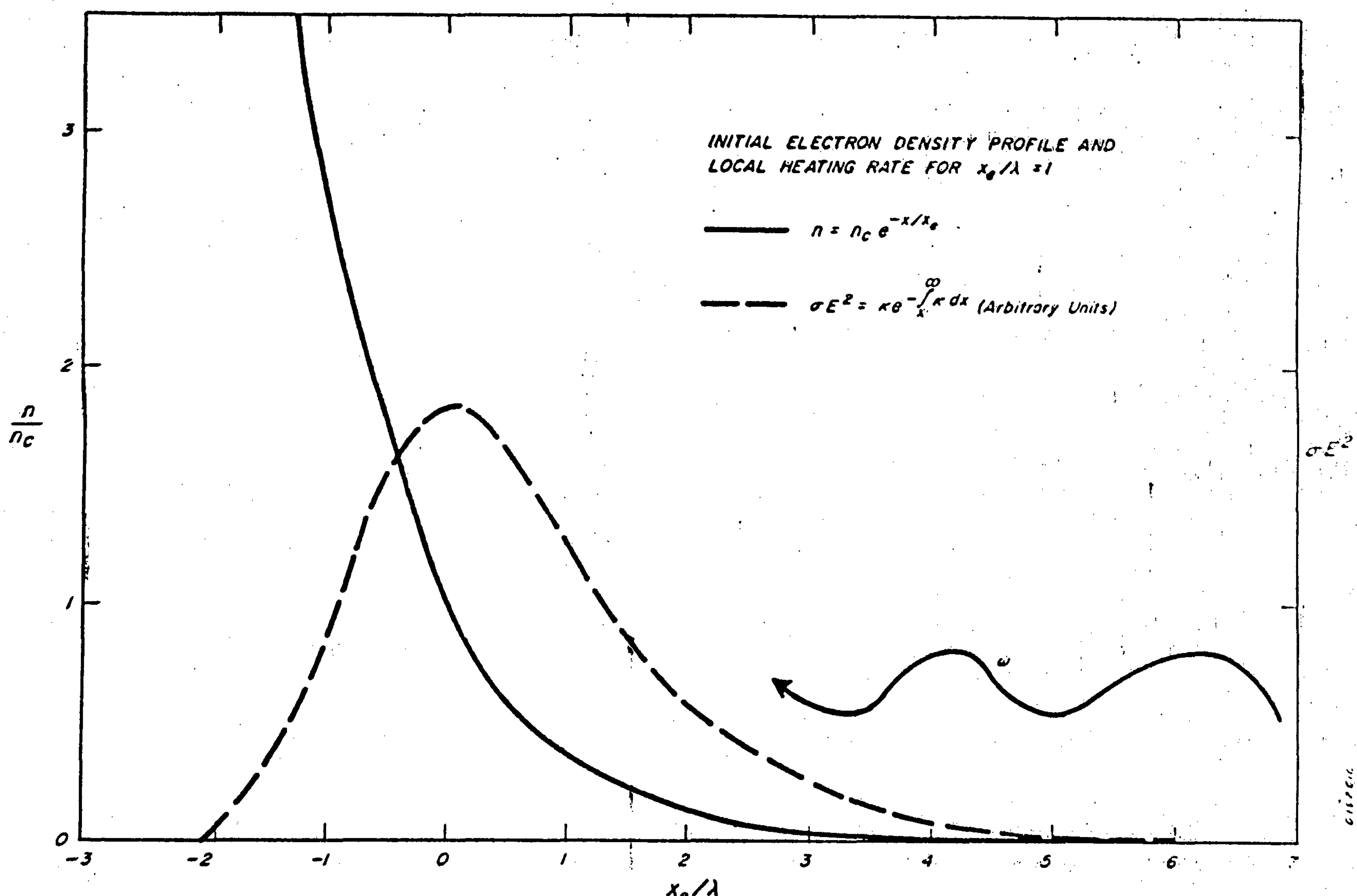

Figure 1. Irradiated Plasma Model (Uncl.)

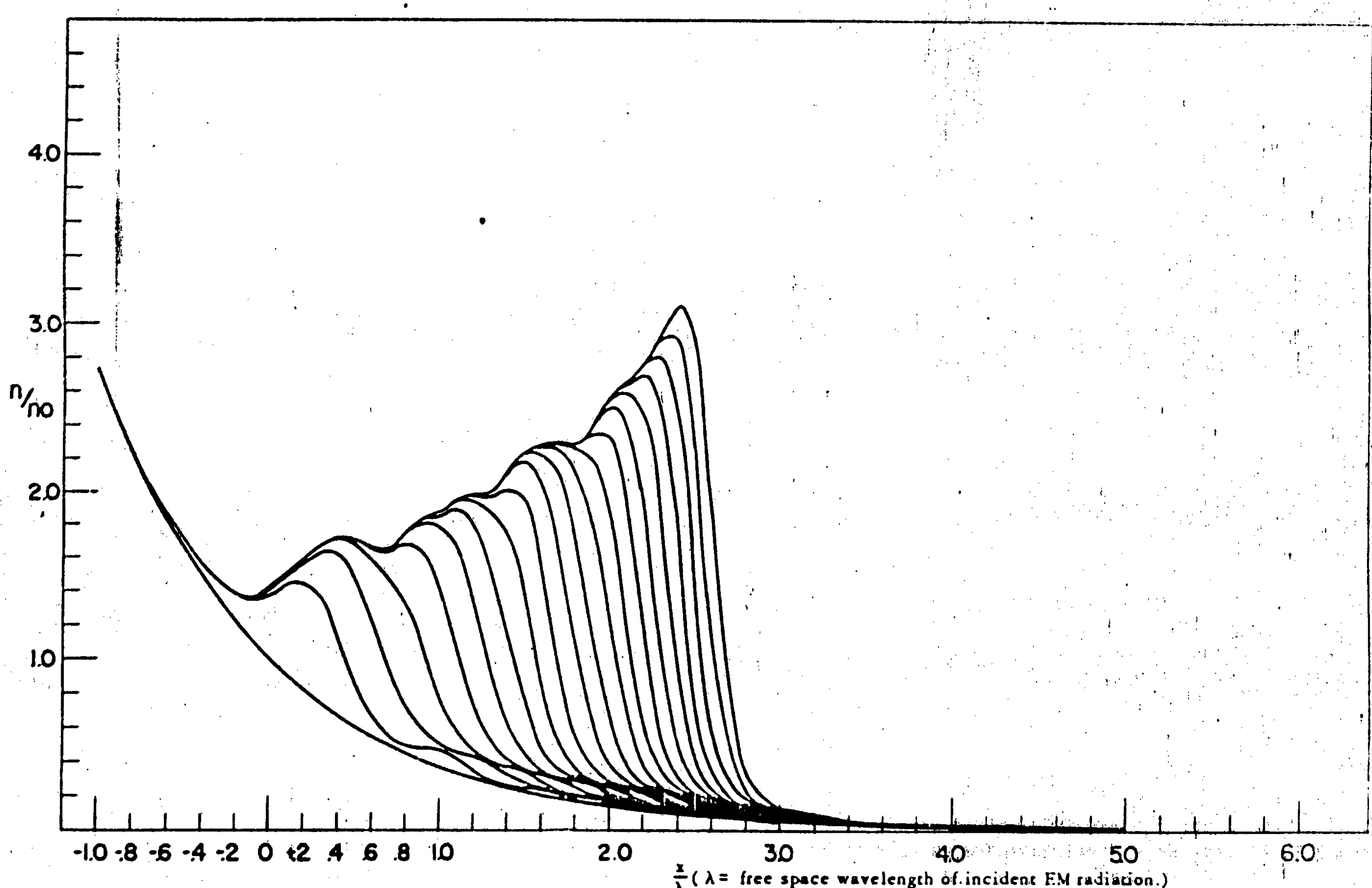

Figure 2. Electron Density Distribution as a Function of Plasma Depth, X, and, Time, T. Each iteration represents a time increment of approximately eighteen micro-seconds. ($\frac{\nu}{\omega}$ = 0.1) (Uncl.)

tors if and only if the medium parameters (permeability, index of refraction, conductivity) are independent of time and position. In our model, on the other hand, the time variation of the conductivity, through its dependence on the electron density, is the heart of the problem. (Uncl.)

Our procedure is as follows:

a. Express the space-time dependence of the heating rate using the WKB approximation.

b. Use kinetic theory to relate the heating rate to the local electron growth.

c. Develop the first-order partial differential equation which must be satisfied by the steady-state upstream progressive wave.

d. Use this constraint to determine the integro-differential equation expressing the electron density as a function of position, and implicitly of time.

e. Obtain the general solution of the equation and the specification of the physical parameters. (Uncl.)

II. THEORY

A. WKB Approximation

Figure 1 shows our model. A plane monochromatic beam of radian frequency ω progressing in the positive x-direction is incident normally on a plane-parallel inhomogeneous plasma ($\partial/\partial y$, $\partial/\partial z = 0$). Initially the plasma is thermally ionized with the low degree of ionization ($\sim 10^{-3}$) associated with temperatures in the 10^3 - 10^4 °K range. In the figure the initial electron distribution increases exponential to the right according to the prescription

$$n = n_c \, e^{-x/x_e} ,$$

where n is the electron density

$n_c = \frac{m \epsilon_o}{e^2} (\omega^2 + \nu^2)$ is the cutoff density

and x_e is the electron density scaling length. (Uncl.)

Although this electron distribution curve is frequently used because of its mathematical simplicity, the analysis to follow is independent of the initial configuration, requiring only a generally monotonically increasing electron (and temperature) trend. (Uncl.)

If the proportional rate of change of the optical parameters of the medium is small within a wavelength, $\frac{|\nabla k|}{k} \lambda < 1$, the WKB solution of the wave equation is valid. Because of its importance and own intrinsic interest we proceed with a derivation of the WKB solution (Uncl.)

simpler than those cited in the literature.[2] (Uncl.)

The one-dimensional wave equation has the form

$$\frac{d^2E}{dx^2} + k^2(x)E = 0,$$

where k, the complex propagation constant, varies with x. With the substitution of the optical depth

$$d\tau = k\ dx,$$

the equation becomes

$$\frac{d^2E}{d\tau^2} + \frac{1}{k^2}\frac{dk}{dx}\frac{dE}{d\tau} + E = 0.$$

Using the WKB assumption that

$$\left|\frac{1}{k^2}\frac{dk}{dx}\right| = \frac{1}{2\pi}\frac{|k|}{k}\lambda < 1,$$

we find using operator notation that

$$\left(D^2 + \frac{1}{k^2}\frac{dk}{dx}D + 1\right)E = 0.$$

This yields at once

$$D = -\frac{1}{2k^2}\frac{dk}{dx} \pm \sqrt{\frac{1}{4k^4}\left(\frac{dk}{dx}\right)^2 - 1},$$

$$\simeq \pm i - \frac{1}{2k^2}\frac{dk}{dx},$$

(Uncl.)

where we have neglected higher order terms.

Thus the solution becomes

$$E = E_o \exp\left(\pm \int_{x_o}^{x} ik\,dx\right) \exp\left(- \int_{x_o}^{x} \frac{1}{2k}\frac{dk}{dx}\,dx\right)$$

$$= E_o \sqrt{\frac{k_o}{k}} \exp\left(\pm i \int_{x_o}^{x} k\,dx\right)$$

(Uncl.)

To find the local heating rate we use the dispersion relation of the wave equation

$$k^2 = k_o^2 \left(\frac{\epsilon}{\epsilon_o} - \frac{i\sigma}{\omega \epsilon_o} \right)$$

where $k_o = \omega / C$ is the free space wave number,

ϵ / ϵ_o is the square of the index of refraction, and

σ is the plasma conductivity. (Uncl.)

For a medium of high Q, i.e.

$$\frac{1}{Q} = \frac{\sigma}{\omega \epsilon} << 1,$$

we can write approximately

$$k \approx k_o \left(\eta - \frac{i\,\sigma}{2\eta\omega\epsilon_o} \right),$$

$$= \beta - \frac{i\alpha}{2}$$

(Uncl.)

with $\eta = \sqrt{\epsilon/\epsilon_o}$, the index of refraction,

β , the phase constant, and

χ , the absorption coefficient. (Uncl.)

Using these substitutions in the WKB solution we find after some algebra that the local heating rate is given by

$$\sigma E^2 = \sqrt{\frac{\epsilon_o}{\mu_o}}\, E_o^2\, \chi(x)\, e^{\pm \int_{x_o}^{x} \chi\, dx} ,$$

$$= - F_o\, \chi(x)\, e^{\pm \int_{x_o}^{x} \chi\, dx} ,$$

where $\sqrt{\frac{\mu_o}{\epsilon_o}} = 120\pi$ ohms is the intrinsic impedance of free space

and F_o is the incident in vacuo electromagnetic flux. (Uncl.)

The usefulness of the WKB approximation lies in the fact that the ohmic heating rate is directly expressed as a function of the plasma parameters which may within limits be arbitrary functions of position. It is not possible to find the exact solution of the wave equation for the propagation constant varying arbitrarily with space. (Uncl.)

The WKB solution becomes invalid whenever:

1. the propagation constant $|k|$ becomes vanishingly small and/or (Uncl.)

2. at regions of steep gradients where, for example, the electron density changes rapidly. (Uncl.)

The first condition concerns us little. The propagation constant is always finite for a conducting plasma, having the minimum value

$$k_{min} = k_o \sqrt{\frac{\nu}{\omega}},$$

where ν is the collision frequency of an electron with neutral particles. Moreover, as we shall see, only a small portion of the radiation penetrates to this depth. (Uncl.)

The second restriction is more stringent. Since the WKB assumption implies zero reflection, our numerical result will be quantitatively in error whenever the electron density profile no longer satisfies the stipulation that

$$\frac{|\nabla n|}{n} \lambda < 1 . \qquad \text{(Uncl.)}$$

B. Plasma Constitutive Relations

Electromagnetic theory can do no more than to express the dependence of field quantities on the optical properties of the medium. To determine the index of refraction $\sqrt{\epsilon/\epsilon_o}$ or the conductivity we must have recourse to a constitutive relation or "law" (Ohm's law, Snell's law, Kirchhoff's law) of an empirical nature or derived from some model.

Thus in the equation

$$\sigma E^2 = F_o \chi \, e^{-\int_{x_o}^{x} \chi \, dx}$$

we must relate the local heating rate σE^2 to the absorption coefficient χ. (Uncl.)

The most general approach would be via the Boltzmann kinetic equation for the electron velocity distribution function

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \nabla f - \frac{\underline{F}}{m} \cdot \nabla_v f = -S ,$$

where included in $\underline{F}$ are all "action at a distance" forces of gravitational, electrostatic, magnetostatic and electromagnetic origin while S, the collision integral, represents the effects of contact forces. The collision integral can be written as the sum

$$S = S_k^{el} + S_k^{inel} + S_i + S_e , \qquad \text{(Uncl.)}$$

representing contributions from elastic and inelastic electron-neutral particle collisions, electron-ion, and inter-electron collisions, respectively. (Uncl.)

In addition for our problem in which the number of electrons is not conserved the effect of electron creation and destruction through ionization and recombination must be accounted for in the collision integral. (Uncl.)

We can conclude that any approach starting from the complete Boltzmann equation is exceedingly difficult. Fortunately, the conditions set forth in our problem are met by the model known as the Lorentz electron-molecule gas. This model assumes:[3] (Uncl.)

1. The motions of molecules and ions determine the mechanical bulk properties of the gas. This follows from the mass disparity of the electrons and ions-neutral particles. (Uncl.)

2. The motion of electrons and ions determine the electrical properties. In addition, the electrons, being lighter, are more mobile, and in many cases they alone determine the electrical properties of the medium, especially in the case of electrical fields of rapid time variation. (Uncl.)

3. The gas is only slightly ionized ($n \ll N$). We can neglect charged particle interactions. The independent electron model of a gas is obtained. Each of these electrons circulates independently in a "sea" of neutral molecules. (Uncl.)

4. The mass ratio ($m/M \sim 10^{-3} - 10^{-4}$) is small. This gives rise to the observed fact that in a plasma heated by external RF power that electron temperature may be several orders of magnitude ($\sim 10^3$) higher than the neutral particle background. This comes about from two effects. The electrons, being lighter, are more easily heated by the electromagnetic beam. Once heated, because of the mass disparity, the energy transfer per collision ($\sim m/M$) is small. Thus the plasma behaves as a hot electron trap, the electrons being radiatively heated but denied any collisional thermal relaxation. (Uncl.)

Although logically the Boltzmann approach is to be preferred, the difficulties in evaluating the collision integrals on the right side rule this out. Frequently, the generalized Ohm's law is derived from the Boltzmann equation using a "dynamic friction" expression on the right. This rather formidable technique yields the identical answer that a much simpler analysis utilizing the Langevin equation does.[4] (Uncl.)

The equation of motion of an electron driven by an impressed alternating $\underline{E}$ field and colliding randomly with neighboring neutral molecules is

$$m\ddot{\underline{\xi}} = -e\,\underline{E}\,e^{i\omega t} + m\,\underline{A}\,(t) ,$$

where $m\,\underline{A}(t)$ is a stochastic force caused by collisions with the neutral particles.

Since $\underline{A}$ (t) is fluctuating randomly, it cannot be specified at any particular instant. However, certain time averages are known. Thus it can

(Uncl.)

be shown from momentum and energy conservation considerations that

$$\overline{\underline{A}} = -\nu \dot{\underline{\xi}} ,$$

and

$$m \overline{\dot{\underline{\xi}} \cdot \underline{A}} = -\delta\nu \frac{3}{2} k (T_e - T) ,$$

where ν is the collision frequency,

$\delta \sim 2m/M$ is the fraction of electron energy transferred to a neutral particle at a collision,

and $\frac{3}{2} kT_e$, $\frac{3}{2} kT$ are the kinetic energies of the electron and neutral particle, respectively. (Uncl.)

The time-averaged Langevin equation can be considered as the first velocity moment of the Boltzmann equation which yields, of course, the momentum transfer or macroscopic force density equation. Thus

$$m \int \frac{\partial f}{\partial t} v_i \, d\underline{v} + m \int v_j \frac{f}{x_j} v_i \, d\underline{v} - \int e E_j \frac{\partial f}{\partial v_j} v_i \, d\underline{v} = m \int \left(\frac{\partial f}{\partial t}\right)_{coll} v_i \, d\underline{v} ,$$

where we have assumed the only external force acting on the electron is due to the $\underline{E}$-field of an impressed electromagnetic wave. Thus we will neglect the magnetic field of the wave since $\underline{B} \sim (v/c)\underline{E}$ as well as any induced field caused by the reaction of the medium. (Uncl.)

Performing the integration over velocity space and using the continuity equation, we find

$$nm\left(\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j}\right) = -\frac{\partial S_{ij}}{\partial x_j} - ne\,E_i - nm\nu u_i \quad ,$$

where S_{ij} is the pressure tensor. In the collision term on the right side we have made use of the time average of the stochastic force. For an isotropic distribution $S_{ij} = p\delta_{ij}$, we can write the equation in vector form

$$nm\left(\frac{\partial \dot{\underline{\xi}}}{\partial t} + \dot{\underline{\xi}}\cdot\nabla\dot{\underline{\xi}}\right) = -\nabla p_e - ne\underline{E}\,e^{i\omega t} - nm\nu\,\dot{\underline{\xi}} \,. \quad \text{(Uncl.)}$$

If we can ignore the second order space derivative terms ($\partial/\partial x_i = 0$), this reduces to the Langevin equation

$$m\,\ddot{\underline{\xi}} + m\nu\,\dot{\underline{\xi}} = -e\,\underline{E}\,e^{i\omega t} \,. \quad \text{(Uncl.)}$$

The transient response (complementary solution) of this equation is

$$\dot{\underline{\xi}} = \dot{\underline{\xi}}_o\,e^{-\nu t} \,.$$

Thus the relaxation time of the electron is the reciprocal of the collision frequency. This indicates that it takes the electron only one collision to thermalize, i.e. to "forget" its ordered motion and to acquire the kinetic temperature T_e. (Uncl.)

The steady state or particular solution of the Langevin equation which is acquired in a time on the order of $1/\nu$ is

$$\dot{\underline{\xi}} = \frac{-e\underline{E}/m}{i\omega + \nu} .$$

This expression yields for the total current

$$\underline{J} = -ne\dot{\underline{\xi}} ,$$

$$= \frac{ne^2\underline{E}}{m} \frac{(-i\omega + \nu)}{(\omega^2 + \nu^2)}$$

$$= \left[i\omega(\epsilon - \epsilon_o) + \sigma\right] \underline{E} .$$

Thus the permittivity and conductivity of the plasma are given by

$$\frac{\epsilon}{\epsilon_o} = 1 - \frac{\omega_p^2}{\omega^2 + \nu^2} , \quad \frac{\sigma}{\omega\epsilon_o} = \frac{\nu}{\omega}\frac{\omega_p^2}{\omega^2 + \nu^2} ,$$

with $\omega_p^2 = ne^2/m\epsilon_o$. (Uncl.)

To obtain the energy or heat transfer equation, we multiply the Langevin equation by $\dot{\underline{\xi}}$ and average over time using the second stochastic relation. Thus we find

$$m\dot{\underline{\xi}} \cdot \ddot{\underline{\xi}} = -e\dot{\underline{\xi}} \cdot \underline{E} + m\dot{\underline{\xi}} \cdot \underline{A}(t) ,$$

$$\frac{d}{dt}\, n\frac{m}{2}\dot{\underline{\xi}}^2 = \underline{J}\cdot\underline{E} + nm\overline{\dot{\underline{\xi}}\cdot\underline{A}} ,$$

$$\frac{3}{2}nk\frac{dT_e}{dt} = \sigma E^2 - \delta\nu\frac{3}{2}nk(T_e - T) . \quad \text{(Uncl.)}$$

This equation has the simple interpretation that the energy absorbed from the beam (ohmic heating) goes into raising the electron temperature and into heating by collisions the neutral particle background. (Uncl.)

We have seen earlier that the electrons acquire their high random thermal velocity very rapidly ($t \sim 1/\nu$). This electron temperature resulting from the impressed field is found by setting $dT_e/dt = 0$, leading to

$$\frac{T_e}{T} = 1 + \frac{2\sigma E^2}{3nkT\delta\nu} = 1 + \frac{2e^2E^2}{3kTm\delta(\omega^2+\nu^2)}$$

where we have used the constitutive relation derived earlier for the conductivity σ. The form of this equation has led Ginzburg and Gurevich[5] to define a characteristic "plasma field"

$$E_p = \sqrt{\frac{3}{2}kT\frac{m}{e^2}\delta(\omega^2+\nu^2)} \ .$$

Obviously for $E \gg E_p$, we have

$$\frac{T_e}{T} \approx \frac{E^2}{E_p^2} \ . \quad \text{(Uncl.)}$$

For large impressed fields the electron temperature can be a factor of $10^3 - 10^4$ larger than that of the neutral gas, as reported in gas discharge experiments.[3] (Uncl.)

Without the impressed electromagnetic field, we find

$$\frac{T_e - T}{(T_e - T)_{t=o}} = e^{-\delta \nu t} ,$$

so that the relaxation time of the elevated electron temperature T_e is

$$t \sim (1/\delta \nu) . \quad \text{(Uncl.)}$$

The following qualitative picture emerges of the plasma-RF interaction. Very quickly after the initial irradiation ($t \sim 1/\nu$) the electrons acquire the temperature

$$T_e = T \frac{E^2}{E_p^2} .$$

A much longer period, $t \sim (1/\delta\nu)$, is required to transfer the energy of the heated electrons to raise the background temperature of the molecules. It is fortunate that such a time disparity exists. It enables us to treat separately the heating of the electron gas and the neutral particle background. (Uncl.)

Thus the effect of the RF power is to heat the electrons and, more slowly, the neutral gas. If this were all, there would be no dynamic plasma response since simple heating does not affect the electrical properties of the plasma. These depend only on the electron density and the collision frequency. Although the latter is in general a function of temperature, in this first order treatment we assume a constant. (Uncl.)

We began initially, however, with a thermally ionized plasma. Any heating will increase the degree of ionization with a portion of the impressed energy being absorbed in the ionization process (we neglect recombination). Thus we can write:

$$\text{impressed energy} = \text{gas heating} + \text{ionization energy}$$

$$\sigma E^2 = (1 - f)\,\sigma E^2 + f\,\sigma E^2 ,$$

where f is the probability of an ionizing collision. Figure 3 drawn largely from Brown[6] displays the ionization probability for a number of gases where $f = Q_i/Q$, the ratio of the ionization cross-section to the total collision cross-section. (Uncl.)

Equating the ionization energy terms we have, at last, the fundamental equation expressing the space and time dependence of the electron density

$$\frac{\partial n}{\partial t} = \frac{f}{\mathcal{E}_I}\,\sigma E^2 = \frac{f}{\mathcal{E}_I}\, F_o \chi\, e^{-\int_{x_o}^{x} \chi\, dx}$$

(Uncl.)

C. Problem Solution

Making use of the approximate relation for the absorption coefficient

$$\chi = \frac{2\pi}{\eta\lambda}\,\frac{\nu}{\omega}\,\frac{n}{n_c} ,$$

derived earlier, the equation can be expressed in the single dependent

(Uncl.)

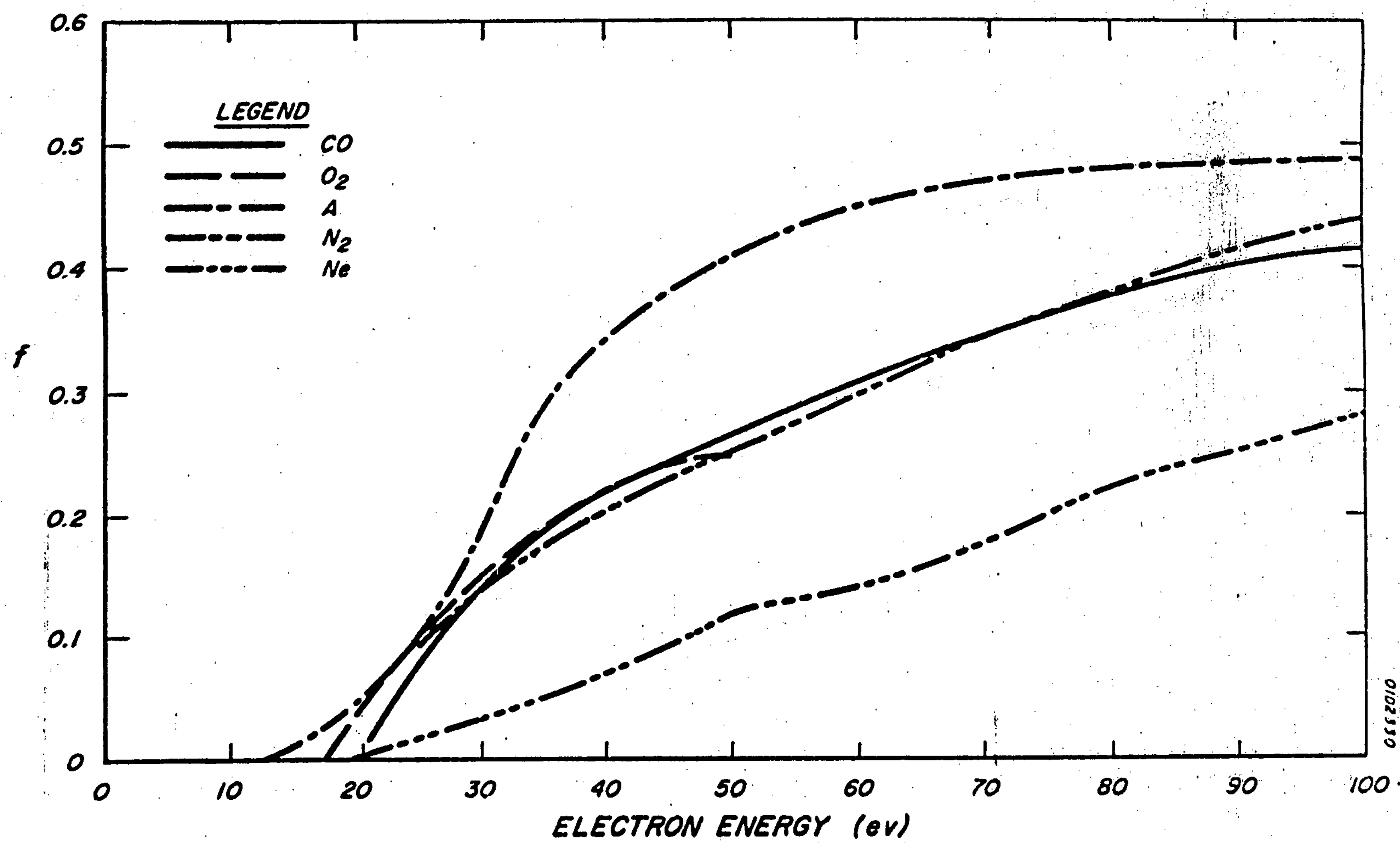

Figure 3. Ionizing Collision Probability vs. Electron Energy (Uncl.)

variable n. Thus

$$\frac{\partial n}{\partial t} = \frac{f}{\mathcal{E}_r} F_o \frac{2\pi}{\eta\lambda} \frac{\nu}{\omega} \frac{n}{n_c} e^{-\int_{x_o}^{\infty} \frac{2\pi}{\eta\lambda} \frac{\nu}{\omega} \frac{n}{n_c} dx} \qquad \text{(Uncl.)}$$

By taking the logarithm followed by a partial differentiation with respect to x, we transform this integro-partial differential equation into the following second order nonlinear partial differential equation

$$\frac{\partial^2 n}{\partial x\, \partial t} = \frac{1}{n} \frac{\partial n}{\partial x} \frac{\partial n}{\partial t} + \frac{2\pi}{\eta\lambda} \frac{\nu}{\omega} \frac{n}{n_c} \frac{\partial n}{\partial t} \quad . \qquad \text{(Uncl.)}$$

It is perhaps surprising that for such an unpromising equation we shall be able to derive the general solution $n = n\,(x,\, t)$ in closed form. We shall find that this solution represents an initial transient response followed by a plasma shield propagating upstream towards the source with constant velocity. (Uncl.)

By the use of the integrating factor n^{-1}, we are able to integrate with respect to time. This yields

$$\frac{1}{n} \frac{\partial n}{\partial x} = \frac{2\pi}{\eta\lambda} \frac{\nu}{\omega} \frac{n}{n_c} + g(x) \; .$$

We determine $g(x)$ by requiring that at $t = 0$,

$$n(x,\, 0) = n_c \; e^{-x/x_e} \quad . \qquad \text{(Uncl.)}$$

This particular initial distribution is chosen because of its mathematical convenience and great flexibility. For example, the electron density scale length is simply the distance x_e. With this boundary condition we now have

$$\frac{1}{n}\frac{\partial n}{\partial x} = \frac{2\pi}{\eta\lambda}\frac{\nu}{\omega}\left(\frac{n}{n_c} - e^{-x/x_e}\right) - \frac{1}{x_e} . \quad \text{(Uncl.)}$$

This equation enables us to determine a very important quantity, the limiting velocity of the plasma shield. Returning to the original integro-differential equation, we can now evaluate the integral in the exponential function for $x \gg x_e$. Thus

$$-\int_x^\infty \frac{2\pi}{\eta\lambda}\frac{\nu}{\omega}\frac{n}{n_c}\,dx = \int_0^{n/n_c} \frac{2\pi}{\eta\lambda}\frac{\nu}{\omega}\frac{n}{\partial n/\partial x}\,d\left(\frac{n}{n_c}\right) = \ln\left(1 - \frac{2\pi}{\eta}\frac{\nu}{\omega}\frac{x_e}{\lambda}\frac{n}{n_c}\right).$$

(Uncl.)

The time derivative for the electron density is therefore

$$\frac{\partial n}{\partial t} = \frac{f}{\mathcal{E}_I} F_o \frac{2\pi}{\eta\lambda}\frac{\nu}{\omega}\frac{n}{n_c}\left(1 - \frac{2\pi}{\eta}\frac{\nu}{\omega}\frac{x_e}{\lambda}\frac{n}{n_c}\right) . \quad \text{(Uncl.)}$$

The velocity of the electron density profile for large x follows from the relation

$$v = -\frac{\partial n/\partial t}{\partial n/\partial x} = \frac{f\ F_o}{\mathcal{E}_I\, n_c}\frac{2\pi}{\eta}\frac{\nu}{\omega}\frac{x_e}{\lambda} . \quad \text{(Uncl.)}$$

We defer discussion of this important shielding velocity until later. Here we merely note that v is a constant, which in turn implies (Uncl.)

that the solution for large x is a progressive wave front of invariant shape moving upstream. (Uncl.)

Proceeding with the general solution, we see that the equation for the space derivative of ln n is of the Bernoulli type. The substitution of $z = 1/n$ reduces this to the following linear differential equation

$$\frac{\partial z}{\partial x} - \left(\frac{2\pi}{\eta\lambda}\,\frac{\nu}{\omega}\,e^{-x/x_e} + \frac{1}{x_e}\right) z = -\frac{2\pi}{\eta\lambda}\,\frac{\nu}{\omega}\,\frac{1}{n_c} \cdot \text{(Uncl.)}$$

Using the usual integrating factor technique we find, after some algebra, that the general solution of this equation is

$$\frac{n_c}{n} = e^{\frac{x}{x_e}}\left[1 + e^{-\frac{e^{-x/x_e}}{a}}\,h(t)\right],$$

where h(t) is a function solely of the time. To determine h(t) we make use of the known limiting velocity for large x. The general solution for $x \gg x_e$ reduces to

$$\frac{n_c}{n(x \gg x_e,\ t)} = e^{\frac{x}{x_e}}\left[1 + h(t)\right],$$

since $e^{-\frac{e^{x/x_e}}{a}} \longrightarrow 1$ for $x \gg x_e$. (Uncl.)

Upon taking the time and space derivatives we find

$$- \frac{n_c}{n^2} \frac{\partial n}{\partial t} = e^{\frac{x}{x_e}} h'(t) ,$$

$$- \frac{n_c}{n^2} \frac{\partial n}{\partial x} = \frac{e^{\frac{x}{x_e}}}{x_e} \left[1 + h(t) \right] . \quad \text{(Uncl.)}$$

The negative quotient of these is of course the velocity which, for large x we have previously found to be constant. Thus

$$v = - \frac{x_e h'(t)}{h(t) + 1} ,$$

which integrates at once to

$$h(t) = -1 + c\, e^{-\frac{vt}{x_e}} .$$

To find the constant c, we note from the general solution that $h(0) = 0$, which requires $c = 1$. (Uncl.)

Thus the time and space evolution of the electron density profile for an irradiated plasma having an initial distribution $n = n_c \exp(-x/x_e)$ is given by (Uncl.)

$$\boxed{\frac{n_c}{n(x,t)} = e^{\frac{x}{x_e}}\left[1 - e^{-\frac{e^{-x/x_e}}{a}\left(1 - e^{-\frac{vt}{x_e}}\right)}\right]},$$

with the limiting shielding velocity

$$v = \frac{f\, F_o}{\mathcal{E}_I\, n_c}\,\frac{2\pi}{\eta}\,\frac{\nu}{\omega}\,\frac{x_e}{\lambda} \approx \frac{F_o}{\mathcal{E}_I\, n_c}\,\frac{f\nu}{c}\, x_e \, ,$$

where $f\nu$ is the ionization collision frequency. (Uncl.)

Figures 4, 5 & 6 display growth of the plasma shield as a function of space and time. (Uncl.)

D. Discussion of Results

The most important concept emerging from this treatment is the limiting velocity and its dependence on the plasma and RF parameters. It appears that after the transient response is washed out, a limiting shield velocity emerges which is specified by basic plasma properties. The direct dependence on the collision frequency arises from the fact that only in collisions is energy transferred to drive the sheath outward. The smaller the e-folding length x_e, the steeper the density profile which leads to deeper deposition and hence slower velocity.

(Uncl.)

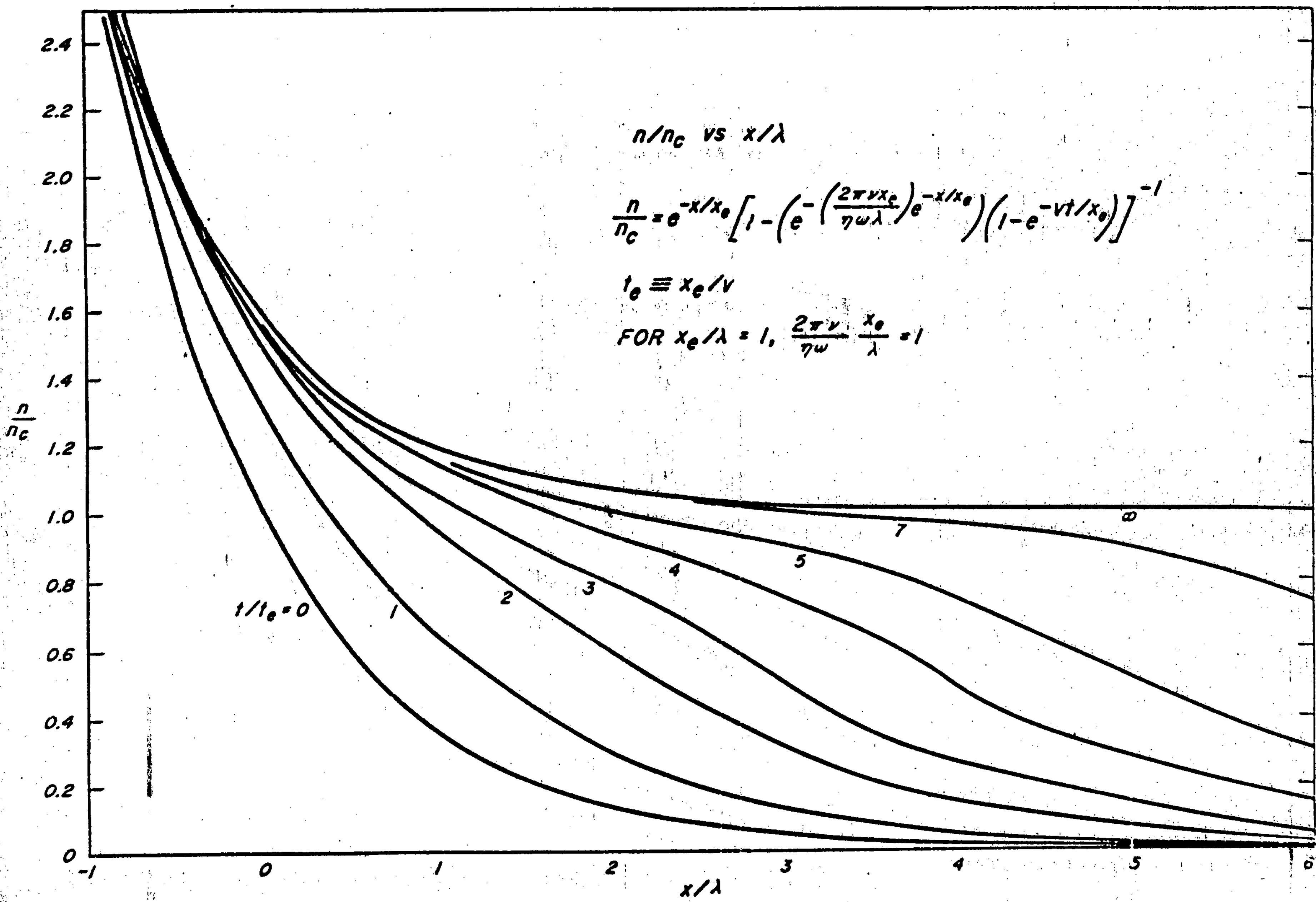

Figure 4. Transient Time Response of the Electron Density of an Irradiated Plasma (Uncl.)

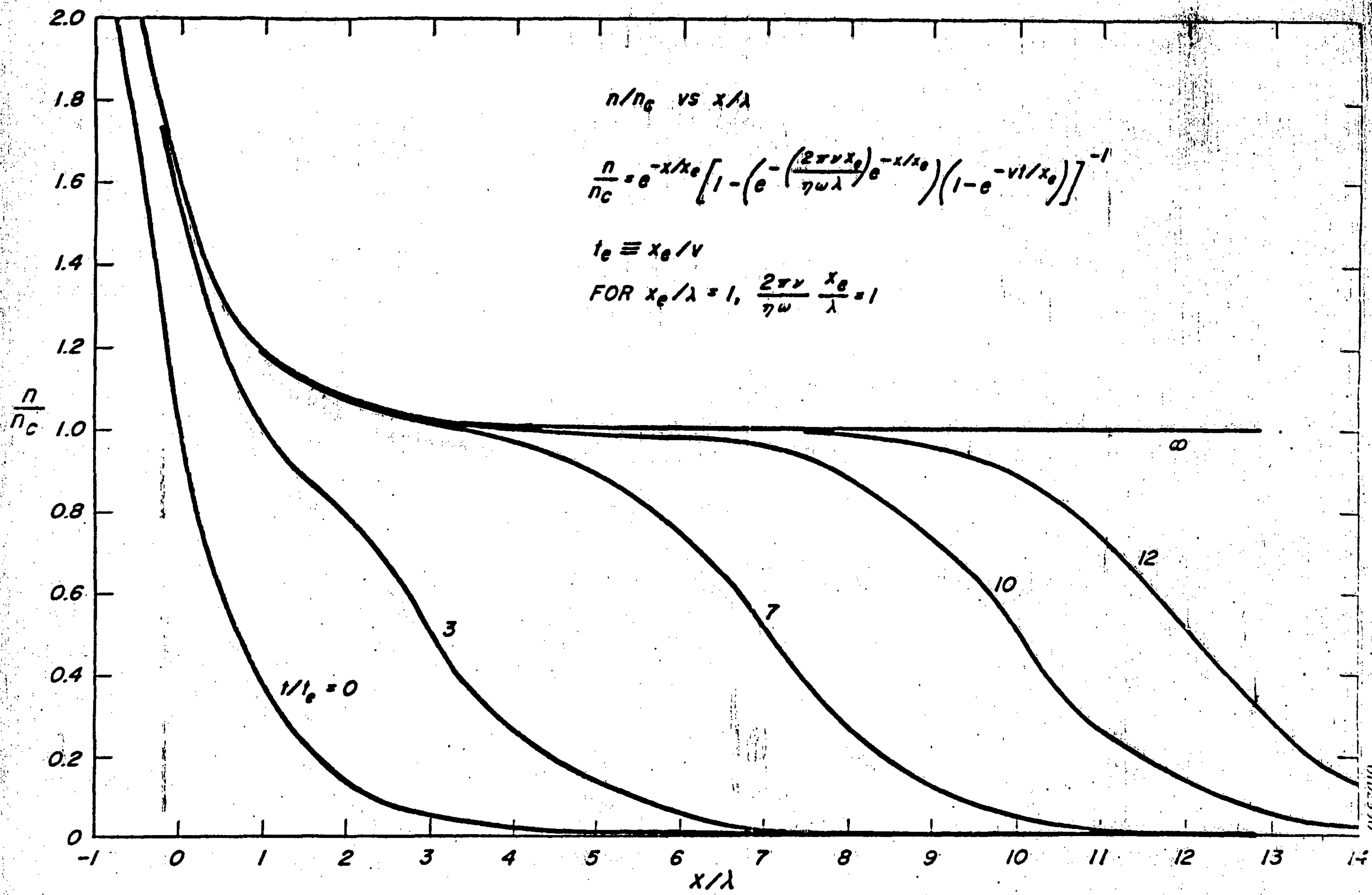

Figure 5. Steady-State Time Response of the Electron Density of an Irradiated Plasma (Uncl.)

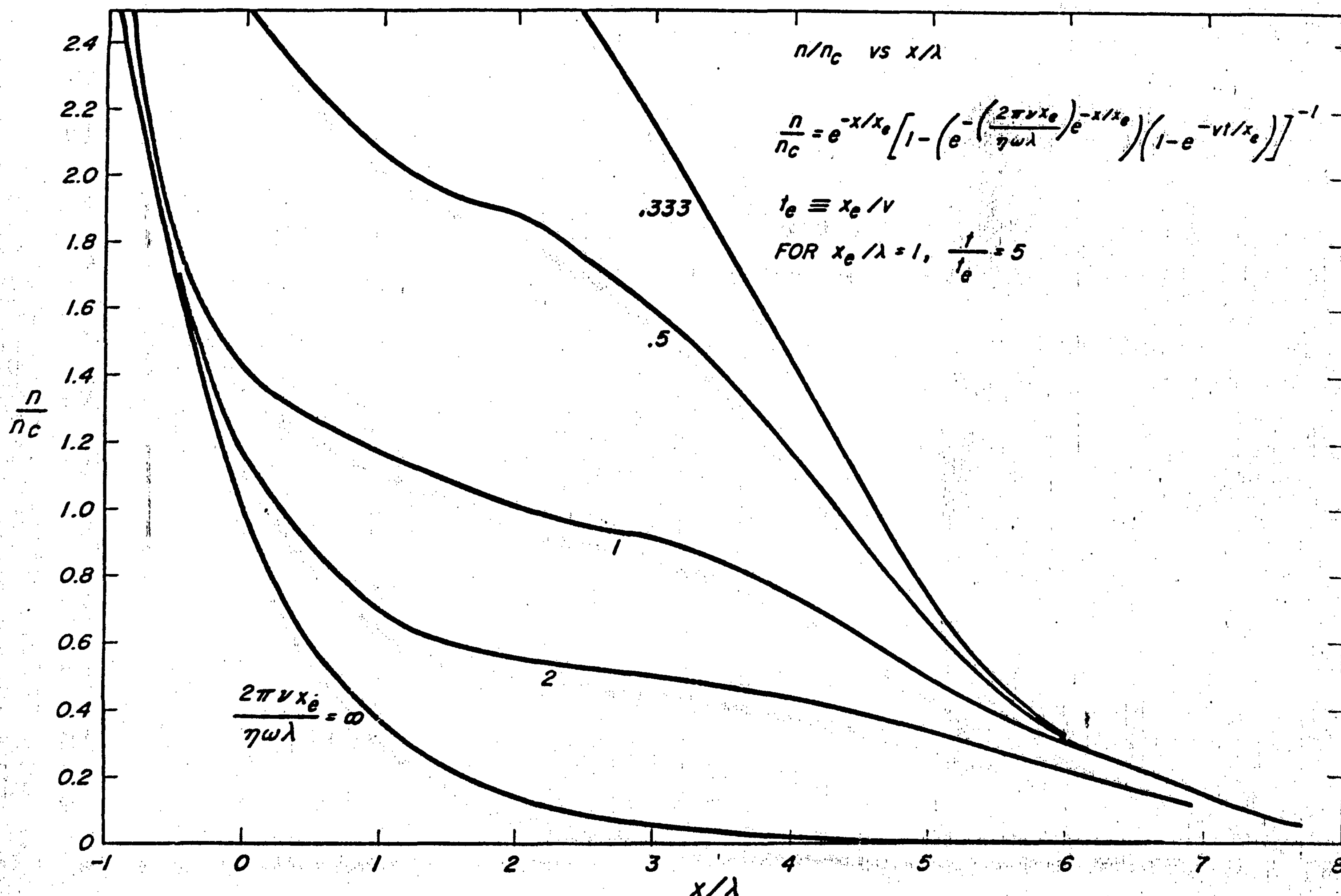

Figure 6. Electron Density Profile of an Irradiated Plasma at a Fixed Time for Various Damping Parameters (Uncl.)

The limiting density profile is easily found from the general solution to be

$$\frac{n_c}{n} = \frac{2\pi}{\eta}\frac{\nu}{\omega}\frac{x_e}{\lambda} + e^{\frac{x - v t}{x_e}}$$

so that the limiting upper density bound is

$$n_{max} = \frac{n_c}{\frac{2\pi}{\eta}\frac{\nu}{\omega}\frac{x_e}{\lambda}} .$$

Figure 7 is a plot of the limiting profile for various values of the initial e-folding length of the electron density. (Uncl.)

There are at least four directions in which to extend this theory to include:

1. The effect of reflection. Our results are valid only for (λ/x_e) $<$ 1. Since the reflection coefficient R depends on λ/x_e and ν/ω we can, as a first approximation, multiply the incident flux F_o by the reflection coefficient appropriate to the initial electron configuration. The advantages of the WKB-approximation in providing a tractable formalism are so important that it should only be superseded when absolutely necessary. (Uncl.)

2. Electron relaxation effects such as recombination, heat conduction, and plasma radiation.

(Uncl.)

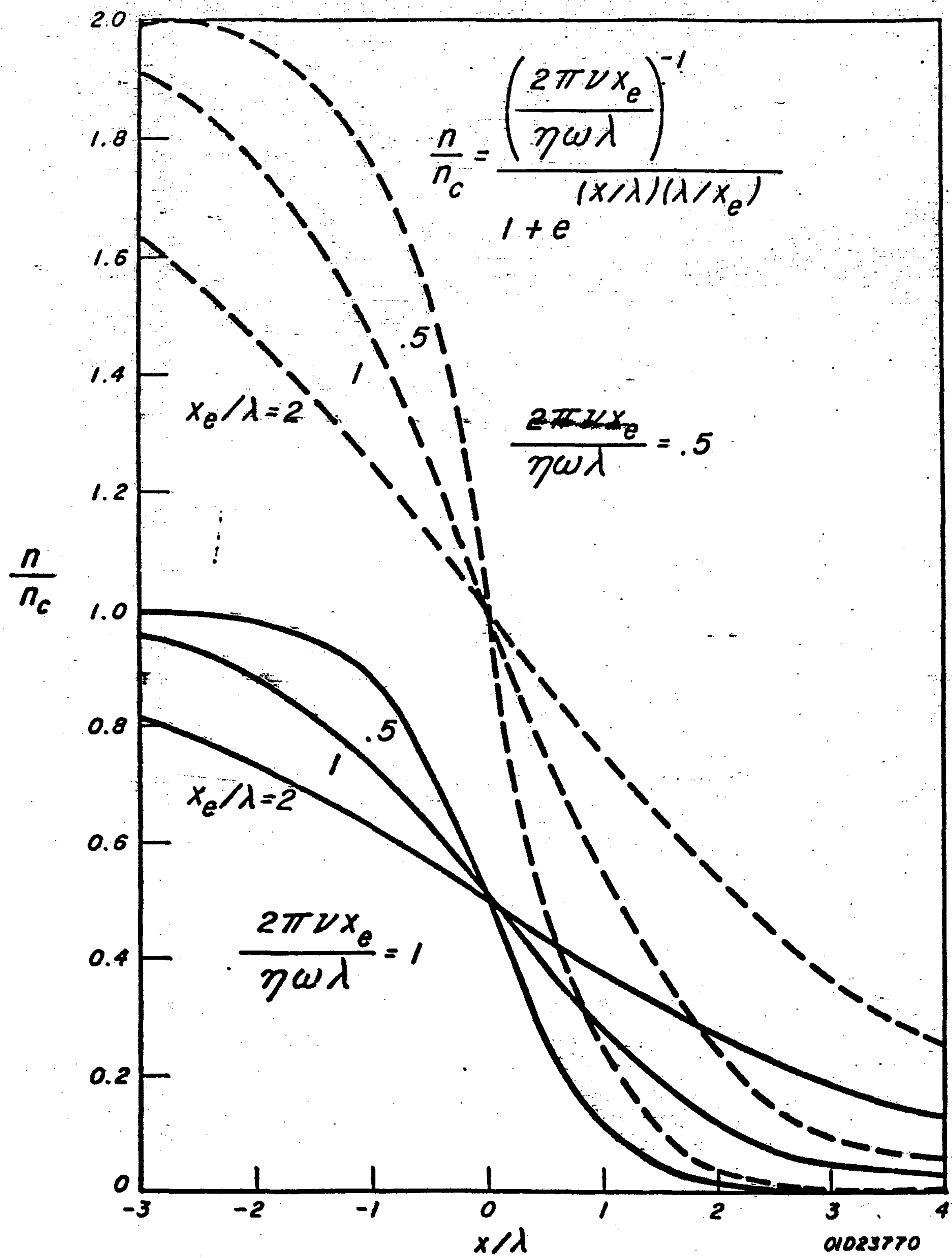

Figure 7. Limiting Shield Profile as a Function of Electron Density Scale Length and of the Damping Parameter. (Uncl.)

3. The effect of the type of initial electron density distribution on the limiting velocity. We will generalize our treatment beyond the undisturbed exponential electron profile. (Uncl.)

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